

CALCULATIONS CONCERNING THE NOISINESS OF A DRIFTING
STREAM OF ELECTRONS

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SUMMARY

A simple argument shows that multivelocity electron flow is not inherently noisy at microwave frequencies despite the random emission of electrons. The reduction of noise due to abrupt acceleration and drift at a constant potential under longitudinal a.c. space-charge fields has been computed for a number of cases.

Electron streams accelerated by potentials of hundreds of volts behave substantially as single-velocity streams. However, the velocity distribution at the cathode, together with shot noise in each velocity class of electrons, gives rise to fluctuations in the velocity and convection current of the stream and to noise in microwave tubes making use of the stream. Haus and Robinson* have shown that the noise figure of a tube may be specified in terms of a parameter $S - \pi$, which is invariant along a single-velocity stream.

In flow suddenly accelerated from a temperature-limited cathode this noise parameter has the value:

$$S - \pi = \frac{kT_c}{2\pi} \quad \dots \quad (1)$$

Here k is Boltzmann's constant and T_c is the temperature of the cathode. In such a stream the noise power is zero.

Siegman, Watkins and Hsieh have recently shown† that, when the electrons are accelerated in space-charge-limited flow in a parallel-plane diode, the noise is less than that given by eqn. (1), and the noise power π is not zero.

A simple argument shows that multivelocity electron flow is in fact not inherently noisy. If electrons of different velocities originate as separate streams and are allowed to drift under space-charge forces until their current fluctuations are changed completely to velocity fluctuations and then combined, the resulting flow has only one sort of fluctuation. This can be removed by velocity-jump deamplification, and so the flow is inherently noiseless.

Further, to some degree the electrons of different velocities could be separated, the above means then being used to reduce the noise parameter; this, however, appears impractical.

As multivelocity electron flow does not inherently lead to noise in amplifiers, there may, however, be various feasible means for deriving from a cathode an electron stream which has a lower noise parameter than that obtained by accelerating the electrons abruptly. We have seen that flow in a space-charge-limited parallel-plane diode is less noisy than that for abrupt acceleration.† It thus seemed likely that a noise reduction would be obtained by accelerating the emitted electrons to a low potential, allowing them to drift at a constant potential, and then accelerating them to the final high potential. This

provides adjustable parameters which may be useful in reducing the noise.

In searching for an accurate and convenient way to make computations for this case, an approach was investigated in which the flow is simulated by a number of discrete single-velocity streams. This formulation is completely consistent and in accord with the distribution-function approach.* In the multistream approach, two equations, one for convection current and one for kinetic voltage, must be integrated, rather than only one equation, that for the distribution function. Further, it turns out that the flow must be divided into an inordinately large number of streams in order to give an accurate simulation of the continuous flow, even for such a simple problem as the noise in abruptly accelerated flow.

Suppose, for instance, we represent the flow by N single-velocity streams, each representing a range of velocities carrying an equal current and each velocity chosen as that which divides a given range into two streams of equal current. We will assume the flow to have been accelerated by about one volt. Table 1

Table 1

Number of streams, N	Noise parameter, N streams
	Noise parameter, continuous flow
10	0.756
20	0.855

shows the ratio of the noise parameter calculated for the flow of N single-velocity streams to the noise parameter calculated for the actual continuous velocity distribution.

Because so many streams were needed to give an accurate result using the multistream approach, it seemed better to use the distribution-function approach, following Siegman, Watkins and Hsieh. J. A. Morrison of the Bell Laboratories adapted this method to the problem and with the help of D. E. Eastwood had a number of computations of noise parameters carried out on a computer.

In such computations the distribution function is computed for certain discrete velocities and the subsidiary variables are obtained by numerical integrations using Simpson's rule. Test computations made using 24 velocities were compared with computations using 50 velocities. As the latter led to a few per cent less noise, further computations were carried out using 50 velocities.

In these computations, two dimensionless parameters are inherent, together with a dimensionless variable representing distance. These are:

$$\eta_0 = \frac{eV}{kT_c} \quad \dots \quad (2)$$

* HAUS, H. A., and ROBINSON, F. N. H.: 'The Minimum Noise Figure of Microwave Beam Amplifiers', *Proceedings of the Institute of Radio Engineers*, 1955, 43, p. 981.

† SIEGMAN, A. E., WATKINS, D. A., and HSIEH, HSUNG-CHENG: 'Density-Function Calculations of Noise Propagation on an Accelerated Multivelocity Electron Beam', *Journal of Applied Physics*, 1957, 28, p. 1138.

* SIEGMAN, A. E., WATKINS, D. A., and HSIEH, HSUNG-CHENG: 'Density-Function Calculations of Noise Propagation on an Accelerated Multivelocity Electron Beam', *Journal of Applied Physics*, 1957, 28, p. 1138.

$$\Omega = \omega \left(\frac{m}{2kT} \right)^{-1/4} \left(\frac{eJ_0}{m_e} \right)^{-1/2} \dots \dots \dots (3)$$

$$\zeta = \left(\frac{m}{2kT} \right)^{3/4} \left(\frac{eJ_0}{m_e} \right)^{1/2} z \dots \dots \dots (4)$$

Here $-e$ and m are the charge and mass of the electron, V is the voltage by which the electrons have been accelerated, k is Boltzmann's constant, T_c is the cathode temperature, J_0 is the current density, ϵ is the dielectric constant of a vacuum and z is distance.

In the computations a frequency of 3000 Mc/s and a temperature of 1000° K were assumed and reasonable values of J_0 and V were chosen. Then computed results can, of course, be scaled to other combinations of J_0 , V , T_c and ω .

Computations were carried out for the 6 cases shown in Table 2.

Table 2

Case	V_0	J_0	$(S - \pi)_{\min}$	Cm to min
	volts	amp/m ²		
1	1	2×10^3	0.69	2.0×10^{-2}
2	1	5×10^2	0.67	3.5×10^{-2}
3	0.345	2×10^3	0.48	8.9×10^{-3}
4	0.345	5×10^2	0.42	1.6×10^{-2}
5	0.198	2×10^3	0.34	6.1×10^{-3}
6	0.198	5×10^2	0.31	1.05×10^{-2}

In Figs. 1-6 normalized values of the noise parameters S , π and $S - \pi$ are plotted against ζ . S and $S - \pi$ were nor-

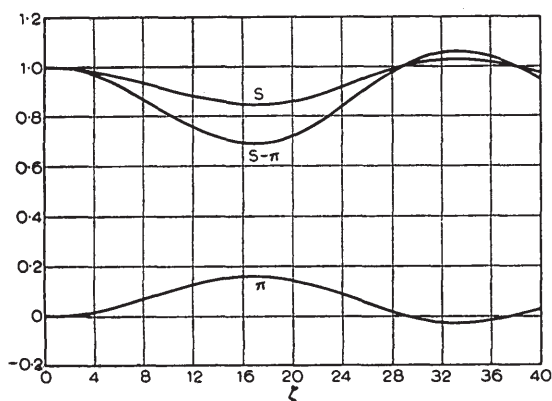


Fig. 1

$V_0 = 1$ volt
 $J_0 = 2 \times 10^3$ amp/m²

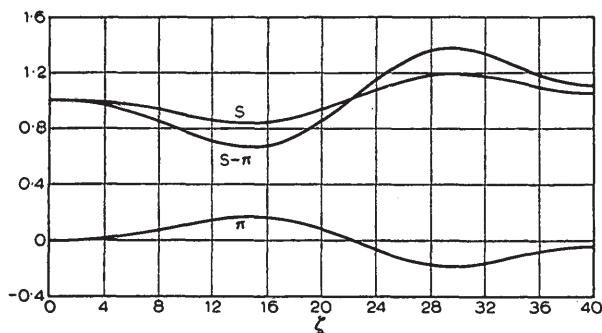


Fig. 2

$V_0 = 1$ volt
 $J_0 = 5 \times 10^2$ amp/m²

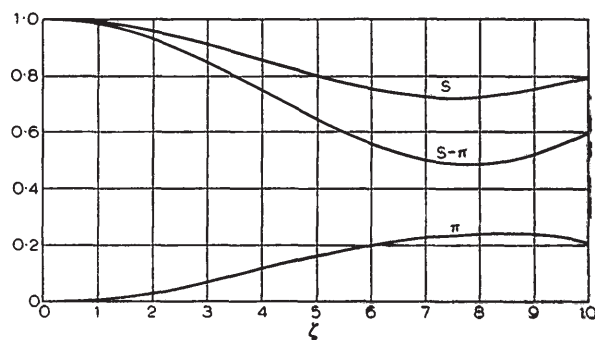


Fig. 3

$V_0 = 0.345$ volt
 $J_0 = 2 \times 10^3$ amp/m²

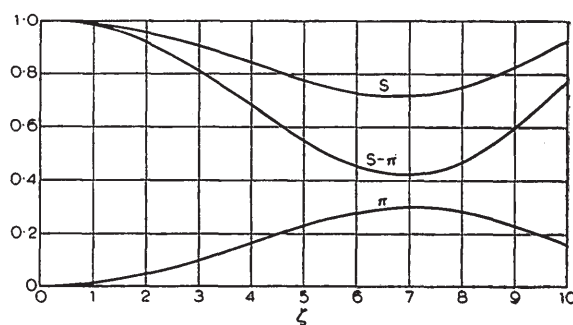


Fig. 4

$V_0 = 0.345$ volt
 $J_0 = 5 \times 10^2$ amp/m²

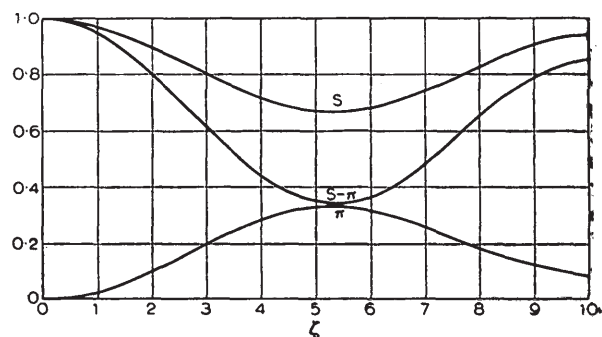


Fig. 5

$V_0 = 0.198$ volt
 $J_0 = 2 \times 10^3$ amp/m²

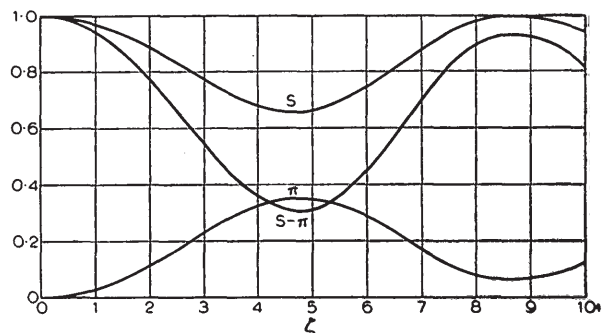


Fig. 6

$V_0 = 0.198$ volt
 $J_0 = 5 \times 10^2$ amp/m²

malized to unity at the cathode, which was taken as emitting electrons with a Maxwellian velocity distribution in a completely random fashion. It is interesting to note that $S + \pi$ is nearly constant with ζ . It can be shown analytically that this must be true near $\zeta = 0$ for a Maxwellian distribution. The minimum values of $S - \pi$ and its position with respect to the cathode

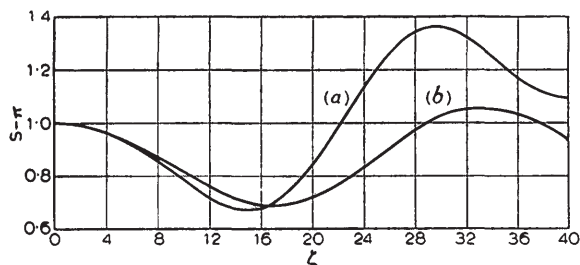


Fig. 7

$V_0 = 1$ volt
(a) $J_0 = 5 \times 10^2$ amp/m²
(b) $J_0 = 2 \times 10^3$ amp/m²

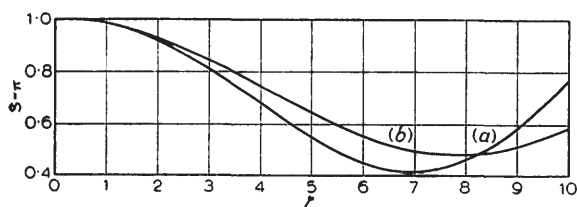


Fig. 8

$V_0 = 0.345$ volt
(a) $J_0 = 5 \times 10^2$ amp/m²
(b) $J_0 = 2 \times 10^3$ amp/m²

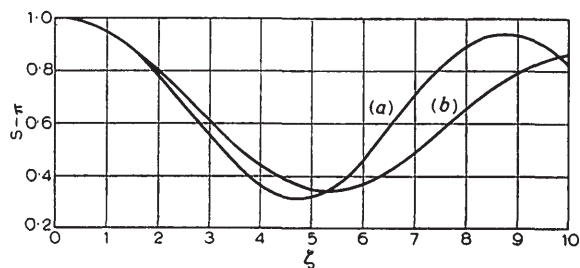


Fig. 9

$V_0 = 0.198$ volt
(a) $J_0 = 5 \times 10^2$ amp/m²
(b) $J_0 = 2 \times 10^3$ amp/m²

are shown in Table 2. Figs. 7-9 compare $S - \pi$ for the same voltage and two different currents.

The minimum attainable noise figure, F_m , is

$$F_m = 1 + \frac{2\pi}{kT}(S - \pi) \quad (5)$$

Here T is the reference temperature, taken as 293° K.

For abruptly accelerated flow the unnormalized value of $S - \pi$ is given by eqn. (1). For abruptly accelerated electrons it is 4.42 times, or about 6.5 dB. When the normalized value of $S - \pi$ is one-third, the corresponding noise figure is 2.14, or 3.3 dB.

Supplementary calculations indicate that $S - \pi$ continues to decrease with decreasing V , so that even lower noise figures are in principle possible. However, we should note that the required drift lengths of Table 1 are so small that it is difficult to see how to realize them in a practical way.

DISCUSSION ON THE ABOVE PAPER

Dr. A. E. Siegman (United States): The author's calculations show correlation developing a short distance beyond the potential minimum. Similar results are obtained by Watkins, Hsieh and myself.* Tien,† on the other hand, has reported Monte Carlo calculations which include the region both before and after the potential minimum. His results are presented for a point far enough beyond the minimum for some correlation to have developed, according to our work. Yet his results do not show any correlation. Why do the two methods of calculation not agree?

Dr. H. Pözl (Austria): Tien calculated for $1.2d_m$, and from Dr. Siegman's curves it can be seen that he should have found a value for Π larger than zero.

Dr. L. M. Field (United States): How does the author reconcile these very short distances at which he seems to observe the minimum of $S - \pi$ with the experimental results of Currie and others, using appreciably longer distances?

Mr. C. R. Russell: Currie and Forster in their paper‡ describing a 3.7 dB noise-figure backward-wave amplifier remark on the very high magnetic fields required. I wonder whether the author would like to comment on this.

Mr. R. C. Glass: Am I correct in assuming the author stated that a multi-velocity beam is inherently noiseless, and that the noise is introduced afterwards by acceleration, high voltage, etc.? If so, the implication would seem to be that, if you do not accelerate the beam in this manner but allow it to drift, it would have no noise at all.

Mr. R. W. Gould (United States): With regard to the discrepancy between the author's results, those of Siegman, Watkins and Hsieh, and Tien's, is it not correct that Tien focused attention primarily on the region between the cathode and the potential minimum and referred his results to the potential minimum or to a plane slightly beyond, whereas both the author and Siegman *et al.* have studied the propagation of the noise beyond the potential minimum? Tien's calculation, in essence, justifies the assumption made by the author about the conditions at the initial plane.

Mr. P. F. C. Burke: Has the author a physical explanation of this reduction of noise in the electron beam drifting at low voltages, and if so, is it in any way similar to the velocity-sorting mechanism he describes as giving a noiseless beam in theory?

Dr. L. M. Field: I may have the explanation for which Mr. Burke is looking, or at least a possible explanation. It was given to me by the author. Streams of different velocities interact with each other if the velocity difference is small compared with their mean velocity. With streams having a very low average velocity, the velocity differences between various streams are large compared with the mean velocity, so that they may be very uncoupled compared with the case where they are all accelerated to a high velocity and their differences are small compared with the mean.

Dr. W. R. Beam (United States): It seems particularly strange that, in Tien's Monte Carlo calculations, different velocity classes of electrons were used, and yet no correlation was observed; on the other hand, no matter what parameters are used by the author, there seem to be tremendous changes in correlation. It is therefore somewhat mysterious that Tien obtained no correlation whatsoever.

* SIEGMAN, A. F., WATKINS, D. A., and HSUNG-CHENG HSIEH: 'Density-Function Calculations of Noise Propagation on an Accelerated Multi-Velocity Electron Beam', *Journal of Applied Physics*, 1947, 28, p. 1138.

† TIEN, P. K., and MOSHMAN, J.: 'Monte Carlo Calculations of Noise near the Potential Minimum of a High-Frequency Diode', *ibid.*, 1956, 27, p. 1067.

‡ *Proceedings of the Institute of Radio Engineers*, March, 1958.

Dr. L. M. Field: In the Table in which distances are quoted, the lower-current beam had its noise figure at a slightly greater distance, as might be expected, since its plasma frequency was slightly lower.

The Tien result has been distinguished from the calculations of Dr. Siegman *et al.* and from those of the author. It would appear that the possibility of getting very low noise by the Tien mechanism may or may not still be true. It is quite independent of this computation which starts with some assumptions at the potential minimum. The author mentions that it is conceivable, since some tubes give a slightly lower noise figure than might be expected from the abrupt acceleration assumed, that the Tien mechanism may be in operation and superimposed on the one described here.

Dr. H. Pötl: Let us consider this multi-velocity electron beam which is split up into N electron beams which have only a very small velocity spread. Δv_i is a measure of the velocity spread in the i th beam after splitting. When these beams travel under space-charge conditions, at the point where the current fluctuation is changed into a velocity fluctuation, the velocity fluctuation arising from Δv_i will have changed into a current fluctuation. The latter will contribute, while summing the beam again, to the total current fluctuation. I cannot see why this resulting current fluctuation should become smaller for increasing N , since we then have an increasing number of decreasing contributions. My argument holds only if no part of the original beam is intercepted.

If we let electrons travel under space-charge conditions at low voltages, the electron encounters may result in a restoration of the Maxwellian distribution and might influence the noise behaviour. Since this influence increases for small electron velocities, it might be a limitation in the case which the author calculated.

Mr. M. T. Vlaardingerbroek (Netherlands): I suggest that the voltage fluctuation of potential minimum will have an effect.

Dr. W. R. Beam: With regard to the author's intuitive description of the 'noiseless' electron gun, the idea of applying thermodynamic theory to electron flow in low-noise electron beams seems very attractive. Recently, however, we have approached some of the leading thermodynamicists in the United States with this complicated problem, and they 'throw up their hands in despair'; but, even though there is no true thermal equilibrium in the beam, there is some sort of thermodynamical process in the gross interaction between the space charges. The author suggests mixing beams of initially zero temperature and zero entropy. This is similar to the thermodynamical problem of the mixing of gases, where the total entropy increases; under certain conditions the temperature can also increase, depending on the dynamics of the mixing. This may be a clue to the electronic interaction process in the initial acceleration region of the electron gun.

Mr. G. Knott (communicated): The author shows, by an ingenious physical argument, that a multi-velocity electron flow is not inherently noisy. This view can also be supported by the following argument.

Consider an electron moving with velocity v between the plates of a parallel plane condenser. A current is induced in the leads, and if the velocity is normal to the plates it is given by

$$i = \frac{ev}{d}$$

where e is the charge on the electron and d is the separation of the plates. If there are n electrons between the plates, the induced current will be made up of n components. Suppose that one plate is the cathode and the other is the anode of a parallel-plane diode. Because of shot-noise fluctuations in the emission, the number of electrons in the cathode-anode space will vary with time, and this will result in a variation in the number of components in the induced current. This variation in the number of components is the induced shot noise in the current.

If we make the cathode-anode distance large, so that the transit time of the electrons is sufficiently long to average out the shot fluctuations in emission, the total number of electrons in the cathode-anode space can be taken as constant. Since each current component depends only on the velocity and not at all on the position of the electrons, the fact that the charge density varies from point to point in the cathode-anode space does not produce any fluctuation in the induced current. The shot noise has now been removed from the induced current and we are left with a kinematic component only. Since, in principle, a single component of noise can be attenuated to zero the electron stream can be said to be inherently noiseless.

Dr. J. R. Pierce (in reply): It is difficult to define the real causes for the discrepancies between the theories. However, the calculations of Tien and Mosham were for a short-circuited diode, whereas Siegman *et al.* assume the total current to be zero. It would be extremely interesting to know how the S and π functions vary in short distances beyond the potential minimum for a short-circuited diode.

In reply to Dr. Field, it is possible that my computed distances for a minimum of $S - \pi$ are smaller than Currie's drift distances for the lowest noise figure. My calculations assume a solid beam with longitudinal fields only, giving strong space-charge forces, while Currie used a very thin hollow beam which would have weaker space-charge forces for the same current. Perhaps the high magnetic fields required by Currie and Forster are required to make the beam very thin.

In reply to Mr. Glass, I believe that the noisiness depends on how we use the beam. In ordinary devices we accelerate it to such a velocity that it acts very nearly as a single-velocity beam of a noisiness which depends on the method of acceleration. Certainly, allowing the beam to drift at a very low potential should not increase its noisiness, but we do not know how to use such a low-voltage beam for the purpose of low-noise amplification.

In reply to Mr. Burke, the sort of noise reduction I describe is dissimilar to velocity sorting.

I have examined Dr. Pötl's objection, and it appears that the total of all sorts of noise is reduced as the number of streams is increased, in the absence of interception.